Annexe 3

ParisTech
INSTITUT DES SCIENCES ET TECHNOLOGIES
PARIS INSTITUTE OF TECHNOLOGY

RECOMMENDED KNOWLEDGE IN MATHEMATICS

1 - ALGEBRA

1.1 Set theory

Operations on sets, characteristic functions.
Maps, injectivity, surjectivity.
Direct and inverse image of a set.
Integer numbers, finite sets, countability.

1.2 Numbers and usual structures

Composition laws; groups, rings, fields.
Equivalence relations, quotient structures.
Real numbers, complex numbers, complex exponential.
Application to plane geometry.
Polynomials, relations between the roots and the coefficients.
Elementary arithmetics (in **Z/nZ**).

1.3 Finite dimensional vector spaces (*)

Free families, generating families, bases, dimension.
Determinant of n vectors; characterization of bases.
Matrices, operations on matrices.
Determinant of a square matrix; expansion with respect to a line or to a column; rank, cofactors.
Linear maps, matrix associated to a linear map.
Endomorphisms, trace, determinant, rank.
Linear systems of equations.

1.4 Reduction of endomorphisms

Stable subspaces.
Eigenvalues, eigenvectors of an endomorphism or a square matrix; similar matrices; geometrical interpretation.
Characteristic polynomial, Cayley-Hamilton theorem.
Reduction of endomorphisms in finite dimension; diagonalizable endomorphisms and matrices.

1.5 Euclidean spaces, Euclidean geometry

Scalar product; Cauchy-Schwarz inequality; norms and associated distances.
Euclidean spaces of finite dimension, orthonormal bases; orthogonal projections.
Orthogonal group O(E); orthogonal symmetries.

Orthogonal matrices; diagonalization of symmetric real matrices.
Properties of orthogonal endomorphisms of $\mathbf{R}^2$ and $\mathbf{R}^3$.

(*) In several countries linear algebra is studied only in $\mathbf{R}^k$ or $\mathbf{C}^k$; the candidates from these countries are strongly advised to get familiar with the formalism of abstract vector spaces.

2 - ANALYSIS AND DIFFERENTIAL GEOMETRY

2.1 Topology in finite dimensional normed vector spaces

Open and closed sets, accumulation points, interior points.
Convergent sequences in normed vector spaces; continuous mappings.
Compact spaces, images of compact sets by continuous mappings, existence of extrema.
Equivalence of norms.

2.2 Real or complex valued functions defined on an interval

Derivative at a point, functions of class C^k.
Mean value theorem, Taylor's formula.
Primitive of continuous functions.
Usual functions (exponential, logarithm, trigonometric functions, rational fractions).
Sequences and series of functions, simple and uniform convergence.

2.3 Integration on a bounded interval

Integral of piecewise continuous functions.
Fundamental theorem of calculus (expressing the integral of a function in terms of a primitive).
Integration by parts, change of variable, integrals depending on a parameter.
Continuity under the sign $\int$, differentiation under the sign $\int$.
Cauchy-Schwarz inequality.

2.4 Series of numbers, power series

Series of real or complex numbers, simple and absolute convergence.
Integral comparison criterion, product of absolutely convergence series.
Power series, radius of convergence; function that can be expanded in a power series on an interval.
Taylor series expansion of e^t, $\sin(t)$, $\cos(t)$, $\ln(1+t)$, $(1+t)^a$ where a is a real number.

2.5 Differential equations

Linear scalar equations of degree 1 or 2, fundamental systems of solutions.
Linear systems with constant coefficients.
Method of the variation of the constants.
Notions on non-linear differential equations.

2.6 Functions of several real variables

Partial derivatives, differential of a function defined on $\mathbf{R}^k$.
Chain rule.
C^1-functions; Schwarz theorem for C^2-functions.
Diffeomorphisms, inverse function theorem.
Critical points, local and global extrema.

Plane curves; tangent vector at a point, metric properties of plane curves (arc length, curvature).
Surfaces in $\mathbf{R}^3$, tangent plane to a surface defined by a Cartesian equation $F(x,y,z) = 0$.

Physics Syllabus

The required knowledge syllabus for applicants whose main examination subject is physics is detailed below. For applicants whose secondary examination subject is physics, only Newtonian mechanics of the material point as well as basic facts about Maxwell's equations, including electrostatics, are required.

All applicants should know the numerical values of the basic constants of physics, as well as the orders of magnitude of the physical phenomena of nature.

The applicants should be able to display excellent standard mathematical skills.

I. MECHANICS

Newtonian mechanics

Mechanics of solids

Statics and mechanics of fluids

Applications of mechanics

II. ELECTRIC CIRCUITS

III. ELECTRICITY AND MAGNETISM

Electrostatics

Magnetostatics

Electromagnetic waves

IV. OPTICS

Geometrical optics

Wave optics

V. THERMODYNAMICS

Perfect gas

First and second principles of thermodynamics

Physical constants

The values of Planck, Boltzmann and Avogadro constants, the charge and the mass of the electron, the speed of light in vacuum as well as the electric permittivity and the magnetic permeability of the free space should be known to the applicants (at least two significant digits are required). For numerical applications, the *SI* unit system is recommended.

Orders of magnitude

Physics is an **experimental science** whose purpose is to describe as **quantitatively** as possible the surrounding world. As a result, the applicants should be able to assess the orders of magnitude of the physical effects they are investigating.

The orders of magnitude of quantities such as the magnetic field of the Earth, the radius of the Earth, the acceleration of free fall at the surface of the Earth, the concentration of electrons in a typical metal, the wavelengths of the electromagnetic waves of the visible spectrum, the distance between two atoms in a solid or liquid, the Bohr radius of the fundamental state of the hydrogen atom, the size of the nucleus, are a minimum basis of required knowledge.

Minimal requirements of calculation skills

In order to be able to follow the teaching of natural sciences profitably, the applicants are supposed to master a number of calculation skills, generally taught during the first three years of university studies. These are:

Expansions

It is unlikely, when solving a physics problem, that one will not need to analyze the behaviour of a physical quantity $A(x)$ in the neighbourhood of a particular value of the x variable. The usual expansions

$$if\ x \approx 0 \ \Rightarrow \ \sin x \approx x - \frac{x^3}{6};\ \cos x \approx 1 - \frac{x^2}{2};\ \tan x \approx x + \frac{x^3}{3}\ ;\ \cot gx \approx \frac{1}{x} - \frac{x}{3}$$

$$if\ x \approx 0 \ \Rightarrow \ e^x \approx 1 + x + \frac{x^2}{2}\ ;\ (1+x)^\alpha \approx 1 + \alpha x + \frac{(\alpha)(\alpha - 1)}{2}x^2 ; \ln(1+x) \approx x$$

are therefore a part of the required knowledge basis.

Derivatives and primitives of the functions of a single variable

For a function of the variable x, what is $\frac{df}{dx}\ ;\ \frac{d^2 f}{dx^2}\ ;\ \ldots\ldots\ ;\ \frac{d^n f}{dx^n}$?

Derivatives of the basic functions :

x^n, $\ln x$, e^x, $\sin x$, $\cos x$, $tg\, x$, $\cot g\, x$ as well as $f(g(x))$.

Rules of derivation of the product and the quotient of two functions of a real variable.

Primitives of the basic functions above.

Integration by parts, $\int u dv = uv - \int v du$.

Convergence problems appearing when the integration interval is infinite are normally considered as a part of the mathematical examination. One should only know that, at infinity, the function should decrease faster than $1 / x$ while in the neighbourhood of x_0 the function should not diverge faster than $1 / (x - x_0)$.

Integral transforms (e.g., of Fourier, Laplace or Hilbert) are not a part of the entrance examination syllabus.

Functions of several variables. Usual differential operators.

What does $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$ mean ?

Calculation of the partial derivative with respect to an independent variable in the case of a function of several variables.

The expression of the usual operators (gradient, curl, divergence) is required in rectangular co-ordinates only.

Gradient of a function of the rectangular coordinates.

Let $\vec{\nabla}$ be the gradient operator $\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$ where $\vec{i}, \vec{j}$ and $\vec{k}$ are unit vectors along the respective rectangular axes.

Compute $\vec{\nabla} f$. Conversely, if $\vec{\nabla} f$ is given, compute f.

Curl.

Let $\vec{A}(\vec{r})$ is a vector field. Compute $\vec{\nabla} \times \vec{A}$.

If $\vec{A} = \vec{\nabla} f \quad \Rightarrow \vec{\nabla} \times \vec{A} = \vec{0}$

Divergence.

Let $\vec{A}(\vec{r})$ is a vector field. Compute $\vec{\nabla} . \vec{A}$.

Laplacian and vector Laplacian.

$$\vec{\nabla}^2(f(\vec{r})) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Compute $$\vec{\nabla}^2(\vec{A}(\vec{r})) = \begin{pmatrix} \frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} \\ \frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} \\ \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} \end{pmatrix}$$

Multiple integrals. Stokes, Gauss – Ostrogradski theorems.

Multiple integrals are often reducible in ordinary physics problems to simple integrals because the integrants and surfaces (volumes) involved in the calculations present symmetry properties (most often cylindrical or spherical symmetries). Apart from those simple (yet important) cases, the explicit calculation of a multiple integral is not required.

Circulation of a vector field. What does $\oint_{(C)} \vec{A}.d\vec{l}$ mean ?

$\oint_{(C)} \vec{A}.d\vec{l} = \oiint_{\Sigma} (\vec{\nabla} \times \vec{A})\vec{n}dS$: the circulation of a vector field along a closed curve (contour) *C* is equal to the flow of the curl of that field through the surface ∑ subtended by *C*.

$\oiint_{\Sigma} \vec{A}\vec{n}dS = \iiint_{\Omega} \vec{\nabla}.\vec{A}dV$: the flow of a vector field $\vec{A}$ through a closed surface ∑ is equal to the integral of its divergence over the volume Ω enclosed by ∑.

An important corollary of the above theorem is the Gauss' theorem, of particular usefulness to inverse-square-law fields (namely, gravity and electrostatic ones).

Differential equations

Quite often in physics, one has to deal with first order, $\phi\left(x, y, \frac{dy}{dx}\right) = 0$, or second order, $\psi\left(x, y, \frac{dy}{dx}, \frac{d^2 y}{dx^2}\right) = 0$, differential equations.

No theorems concerning the existence of solutions and their regularity are required at the physics examination.

The solution (including the formal one) of a first order differential equation with separable variables $A(x)dx = B(y)dy$ should be known to the applicants.

Second order linear and homogeneous differential equations with constant coefficients

$$\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + \beta y = 0$$

appear often in physics. Therefore, their solution should be perfectly mastered (characteristic polynomial, nature of solutions, critical damping). The fact that such equations have two linearly independent solutions should be known to the applicants.

Second order linear and inhomogeneous equations with constant coefficients are also found frequently in many problems, for instance, in mechanics ones. Therefore, the applicants should be familiar with forced oscillations and resonance concepts.

The general algebraic theory of coupled linear differential equations (e.g., an exponential of an operator) is not included in the physics syllabus.

Likewise, non-linear differential equations are not a part of the syllabus (unless being of first order and with separable variables).

Equations with partial derivatives

The applicants are supposed to know the general solution $f(x, t)$ of the wave equation

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \quad \Rightarrow \quad f(x,t) = A(x - vt) + B(x + vt)$$

where A and B are two arbitrary functions, as well as its generalization to the three dimensional case.

In relation with the wave equation, the concept of a progressive monochromatic plane wave plays an extremely important role. The fact that :

$\psi(\vec{r},t) = A\exp\left[i\left(\vec{k}.\vec{r} - \omega t\right)\right]$ is a solution of $\Delta\psi == \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$ provided that $\omega^2 = v^2 k^2$

should be known to the applicants. As a result, the concepts of wave vector, wavelength, frequency and period are indispensable pre-requisites.

For the sake of scientific culture and without generally requiring their solution, it is advisable that the applicants know the principal phenomenological laws (Fick's, Fourier and diffusion equations). The underlying idea, namely the energy, mass, etc. balances over an elementary volume is important. In particular, the applicants are supposed to be able to perform such balances in a straightforward manner.

The Schrödinger equation and more generally quantum mechanics are not in the entrance examination syllabus. However, the de Broglie relation $\vec{p} = \hbar\vec{k}$ relating the particle momentum to the wave vector associated with it may be required.

Linear algebra

Frequently a problem in physics requires the diagonalization of a matrix and the calculation of a determinant, etc.

As a result, the calculation of a determinant, the diagonalization of a matrix, the concepts of eigenvalues and eigenvectors of a linear operator should be perfectly mastered by the applicants.

Trigonometry

Ordinary definitions and properties of the basic trigonometric functions (sine, cosine, tangent, cotangent) should be perfectly mastered.

A few trigonometry formulas such as $\cos 2x = \cos^2 x - \sin^2 x$; $\sin 2x = 2 \sin x \cos x$; $\sin \alpha + \sin \beta = 2 \sin [(\alpha + \beta) / 2] \cos [(\alpha - \beta) / 2]$; $\cos \alpha + \cos \beta = 2 \cos [(\alpha + \beta) / 2] \cos [(\alpha - \beta) / 2]$, etc. are met in many branches of physics (e.g., in diffraction and interference topics) and should therefore be known by the applicants.

Fourier series are very useful when analyzing many physical wave phenomena. For a regular enough periodic function $f(x)$ having a period d the existence of the expansion

$$\text{if} \quad f(x+d) = f(x) \quad \Rightarrow \quad f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi x}{d} + b_n \sin \frac{2n\pi x}{d}$$

should be known to the applicants. If a Fourier series expansion is required for the solution of an exercise, the explicit expressions for the coefficients a_n and b_n will be provided to the applicant.

I. MECHANICS

Newtonian mechanics

Lagrange's and Hamilton - Jacobi's formulations of mechanics are not a part of the syllabus. Relativistic mechanics is likewise excluded.

The core of the mechanics syllabus is the fundamental principle of dynamics expressed in an inertial reference frame and, in particular:

Newton's laws: the principle of inertia, the principle of action and reaction, the fundamental equation of dynamics. Galilean relativity. Angular momentum theorem. Kinetic energy theorem. Conservation laws. A system of two particles. Central force motion, bound states, scattering states.

Notions of non-inertial frames and forces referred to as « inertia » forces are pre-supposed (in particular, in the case of linear acceleration and uniform rotation frames).

In many cases, the symmetry of the applied forces suggests the use of cylindrical or spherical co-ordinates. The applicants should know the expressions of the velocity and the acceleration of a material point in cylindrical co-ordinates and should be able to find them, when guided, in the case of spherical co-ordinates.

The concept of potential energy V. $\vec{F} = -\vec{\nabla} V$. If $\vec{F} = -\vec{\nabla} V \quad \Rightarrow \oint_{(C)} \vec{F}.d\vec{l} = V(1) - V(2)$ is independent of the path C followed.

In the case of conservative forces, the conservation of the mechanical energy of an isolated material system should be known to the applicants.

In the case of central forces, $\vec{F} = f(r)\vec{r}$, the conservation of the angular momentum $\vec{L} = \vec{r} \times m\vec{v}$ should be known to the applicants. It makes the solution of Kepler's problem (gravitational or Coulomb's forces in $\frac{\vec{r}}{r^3}$) much easier. The resulting law of areas (Kepler's second law) should be known.

The fact that the trajectories of a Kepler's problem are conic sections (ellipses, circles, parabolas or hyperbolas) should be known. General expressions of the equations of those conic sections are not required in the syllabus (except for a circle, of course). In the case of circular trajectories, the expressions of the potential energy, the kinetic energy and the total energy of a particle with a mass m are required.

The expression of the Laplace force ($q\vec{v} \times \vec{B}$) and the trajectory (a helix) of a charged particle having a charge q in a static and uniform magnetic field $\boldsymbol{B}$ should be known.

The relationship $\frac{d\vec{L}}{dt} = \sum_i \vec{OM}_i \times \vec{F}_i \quad \text{where } \vec{L} = \sum_i \vec{OM}_i \times m\vec{v}_i$ is required because it often presents a means for solving many mechanics problems.

In the case of an isolated system, the conservation of the momentum should be known to the applicants. Elastic or inelastic collision problems should be known. The concept of a centre of mass of a system should be familiar to the applicants. The general theory of collisions and the concept of a scattering cross-section are not a part of the syllabus.

Mechanics of solids

Within this syllabus, the mechanics of solids deals only with rigid bodies (non-deformable solids). Additionally, only solids rotating about a fixed axis are a possible examination topic. Inertia tensors are not required. The expression of the kinetic energy of a solid as a sum of a translational term of its centre of mass and of a rotational term with respect to the centre-of-mass reference frame should be known. The applicants should be able to tackle easily the problem of the compound pendulum (disregarding the fact that the reference frame of the Earth is a non-inertial one).

Statics and mechanics of fluids

Euler's description (the concept of a velocity field) of a fluid. Concepts of flow density, mass flow rate and volume flow rate. The applicants should be able to perform a mass balance. The equation of the conservation of mass in its local form should therefore be known to them.

The definitions of a stationary flow, of an incompressible flow ($\vec{\nabla}.\vec{v} = 0$ in all points), of a non-rotational flow should be known.

Perfect flows: Euler's equation, Bernoulli's relationship on incompressible and homogeneous flows.

In the statics of fluids, the applicants should be able to calculate the resulting force of the pressure forces exerted upon an object (given that, in practice, the result can be expressed in a closed form only if the object is symmetrical enough).

The Archimedes' principle (the buoyancy force applied on an object immersed in a fluid) should be known to the applicants.

Applications of mechanics

Lorentz force, $\boldsymbol{F} = q\ \boldsymbol{E} + q\ \boldsymbol{v} \times \boldsymbol{B}$, (in constant electric and magnetic fields). Linear oscillations; damped harmonic oscillations. Forced oscillations, resonance.

II. ELECTRIC CIRCUITS

Electric voltage. Kirchoff's laws of knots and meshes. Electrical current. Ohm's law. The superposition theorem.

The physical bases of operation of the basic circuit components: resistor, capacitor, induction coil, are required. Their impedances in a sinusoidal regime, R, $-1 / j\omega C$, $j\omega L$ should be known.

The transient regime of charging and discharging a capacitor.

Sinusoidal current and voltage. Maximum value, rms (root mean square) value. Impedances in series add. The inverses of impedances in parallel add.

Study of resonance in circuits in sinusoidal regime. The example of the *RLC* circuit. Relation to mechanical resonance.

III. ELECTRICITY and MAGNETISM

Electrostatics

Coulomb's law. The concept of electric field. Electrostatic field $\boldsymbol{E}$. Circulation and flow of $\boldsymbol{E}$. Gauss' theorem. Symmetry properties of $\boldsymbol{E}$. Electric dipole.

The electrostatic potential ϕ, $\vec{E} = -\vec{\nabla}\phi$, and the Poisson's equation $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ should be mastered by the applicants.

The applicants should know how to calculate $\boldsymbol{E}$ and ϕ when ρ is given in the case of « reasonably simple » charge distributions (in one dimension, for instance). In particular, the distribution of the electrostatic potential between the plates of a planar capacitor should be known.

The concept of electric dipole, the field created by a dipole at a large distance, as well as the interaction energy of a permanent dipole $\vec{d}$ with an electric field $\vec{E}$, $-\vec{d}.\vec{E}$, should be known to the applicants, as well as the definition of the polarization density vector $\vec{P} = \frac{\sum_i \vec{d}_i}{V}$ where V is the volume.

The fact that $\boldsymbol{E} = 0$ in a conductor at equilibrium should be known as well as its corollary that the surface of a conductor at equilibrium is an equipotential one.

Fields in the vicinity of the surface of a metal.

In electrostatics of dielectrics, the fact that the Coulomb's law between two charges q and q' separated by the distance r in a homogenous linear and isotropic dielectric medium is given by $F = \frac{qq'}{4\pi\varepsilon_0 \varepsilon_r r^2}$ where ε_r is the dielectric permittivity should be known to the applicants.

Magnetostatics

Magnetic field $\boldsymbol{B}$. Symmetry properties of $\boldsymbol{B}$.

The magnetic field created by a thin wire carrying a current I : $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3}$ (Biot – Savart's law), the two Maxwell equations $\vec{\nabla}.\vec{B} = 0$; $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$ (Ampère's law), the existence of the vector potential $\boldsymbol{A}$ such that $\vec{B} = \vec{\nabla} \times \vec{A}$ should be known.

The fact that electrostatic potential ϕ and the vector potential $\boldsymbol{A}$ are not unique, while the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ are, should be known. The concept of gauge invariance is not in the syllabus.

Circulation of $\boldsymbol{B}$. The theorem $\oint_{(C)} \vec{B}.d\vec{l} = \mu_0 \sum I$ relating the circulation of the magnetic field on a closed curve (C) to the currents encircled by (C) should be known.

The calculation of $\boldsymbol{B}$ in the case of wires, circular loops, etc. should be mastered by applicants. Fields along the axis of a circular loop and of a coil (solenoid) with a circular cross-section.

Magnetic dipole and magnetic moment $\vec{M}$. The expression of the interaction energy $-\vec{M}.\vec{B}$ of a magnetic moment $\vec{M}$ with a magnetic field $\boldsymbol{B}$ should be known.

The flux of $\boldsymbol{B}$, Φ. Faraday's law, $E^* = -\frac{\partial \Phi}{\partial t}$, electromagnetic induction phenomenon, Lenz' rule.

As in the case of dielectric media, the magnetostatics of magnetic media is not in the syllabus. However, knowledge of the existence of magnetically ordered media (e.g., ferromagnetics) is considered to be a part of the minimal general culture.

Electromagnetic waves

Mis en forme : Gauche, Taquets de tabulation : 1,16 cm,Gauche + 8 cm, Centré

Electromagnetic waves in vacuum:

Maxwell's equations in vacuum. The progressive harmonic plane waves are solutions of these equations. Frequency, wavelength, wave vector. The concept of phase velocity.

In the case of a progressive harmonic plane wave, the electric and magnetic fields are transverse.

The polarization state of an electromagnetic wave. Linear and circular polarizations.

Volume density of the electromagnetic energy and Poynting vector. Group velocity.

The normalization difficulties related to the plane wave solutions should be known to the applicants. The concept of a wave packet as a remedy to these difficulties should be understood. The relationship $\vec{v}_g = \frac{\partial \omega}{\partial \vec{k}}$ should be known.

Electromagnetic waves in matter:

The syllabus is limited to linear and isotropic media where the polarization density of the medium can be put into the form: $\vec{P} = \varepsilon_0 \chi(\omega) \vec{E}$.

The reasoning is performed on the macroscopic fields ***E*** and ***B*** only. It is supposed that these macroscopic fields are averages of the respective microscopic quantities over volumes large with respect to the atomic dimensions, but small with respect to the wavelength.

Maxwell's equations for macroscopic fields should be complemented by relationships valid for the medium where the waves propagate. These constitutive relationships express the relation, linear to the first approximation, between the electric field of the wave and the polarization density ($\vec{P} = \varepsilon_0 \chi(\omega) \vec{E}$) and the current density ($\vec{J} = \sigma \vec{E}$).

When the constitutive relationships are linear, the reaction of the medium to the wave can be expressed as a frequency-dependent dielectric constant $\varepsilon(\omega)$. The fact that $\varepsilon(\omega)$ is generally complex should be known.

For waves of the type $\exp j\left(\omega t - \vec{k}.\vec{r}\right)$ with ω being real and ***k*** complex, the concepts of a complex refraction index, dispersion and absorption should be known to the applicants.

The microscopic models describing the material polarization of the medium are limited to the Drude model and to the model of the elastically bound electron (Lorentz model).

IV. OPTICS

Geometrical optics

The concept of light rays. Reflection and refraction by a plane mirror. Snell – Descartes' laws. Limit angle. Total reflection.

The applicants should know what spherical mirrors and lenses are, as well as the conjugation and magnification relationships.

Wave optics

Reflection and refraction of a harmonic progressive polarized plane wave at the interface between two dielectric media. Proof of Snell – Descartes laws. The amplitude transmission and reflection coefficients will be provided to the applicant if needed during the oral examination.

The concept of an optical path**.** Interference between two totally coherent waves. Michelson's interferometer. Thin slabs. Pérot –Fabry cavity.

The theories of time and space coherences are not in the syllabus. However, the influence of a spectral doublet on the visibility of the fringes of a Michelson's interferometer may be required.

Diffraction at infinity. Huyghens – Fresnel principle. Diffraction by a rectangular slit. Diffraction at infinity by two slits (Young's slits), by a row of slits. The calculation of the diffraction pattern for a circular (or any other more complex form) slit is excluded from the syllabus.

V. THERMODYNAMICS

The usual thermodynamics functions: internal energy, entropy, enthalpy, free energy, free enthalpy, as well as their differentials should be known.

The heat capacities at constant volume and at constant pressure equal the partial derivatives of internal energy and of enthalpy with respect to the temperature.

The definitions of extensive and intensive variables, as well as of thermodynamic equilibrium should be known.

Perfect gas

Perfect monoatomic gas model. Maxwell - Boltzmann distribution of velocities for a monoatomic perfect gas including N identical atoms having a mass m at a temperature T in a volume V. The number of atoms dN located at a distance $\vec{r}$ within d^3r having a velocity $\vec{v}$ within d^3v is :

$$dN = A \exp\left(-\frac{mv^2}{2k_B T}\right) d^3r\, d^3v \quad ; \quad \int_V dN = N$$

where A is a normalization factor. The equipartition theorem results :

$< v_x^2 > = < v_y^2 > = < v_z^2 > = \frac{k_B T}{m}$ as well as $U = \frac{3}{2} N k_B T$ and $C_v = \frac{3}{2} N k_B$.

Collisions against walls. Relationship between pressure and mean square velocity.

Perfect gas in a field of forces having a potential energy $V(\mathbf{r})$.

The number of atoms dN located at a distance $\vec{r}$ within d^3r having a velocity $\vec{v}$ within d^3v is :

$$dN = A \exp\left(-\left(\frac{mv^2}{2k_B T} + V(\vec{r})\right)\right) d^3r d^3v \quad ; \quad \int_V dN = N$$

Barometric formula, $p(h) = p(0) \exp\left(-\frac{mgh}{k_B T}\right)$.

The limits of the perfect gas model. Real gases. The van der Waals gas is often taken as an example. No formulas (internal energy, thermal capacity,…) are required, but the physical reasons for the modification of the perfect gas model into van der Waals state equation should be known to the applicants.

First and second principles of thermodynamics

First principle. Internal energy U. Heat transfer. Work exchanged by a system. The work of pressure forces. $dU = T\,dS - p\,dV$. Enthalpy and Joule – Thomson expansion. Enthalpy of a perfect gas. $C_p - C_v = R$.

Second principle. The entropy S. Entropy balance. Reversible and irreversible processes. Thermodynamic definition of the temperature.

The entropy of the perfect gas. For a condensed phase, it is assumed that $dS = \frac{C_p}{T} dT$.

For the entrance examination, the applicants are only required to know the fact that the entropy of a system and of its environment is either increasing or constant.

Heat machines. Ditherm cycle. Efficiency. Carnot's theorem.

Equilibrium between the phases of a pure substance. Triple point, critical point, enthalpy and entropy of phase changes. Clapeyron's formula.

Free energy and free enthalpy: definitions and differentials. Chemical potential, $\mu = \left(\frac{\partial F}{\partial N}\right)_{T,V}$. The case of the perfect gas. Equilibrium between two phases 1 and 2, $\mu_1(T,P) = \mu_2(T,P)$. Generalization; Gibbs' phase rules.

Updated 26/04/2006